MATHEMATICS vs. THE LAW
Bruce Blackadar

> "Before you enter on the study of law a sufficient ground work must be laid. ... Mathematics and natural philosophy are so useful in the most familiar occurrences of life and are so peculiarly engaging and delightful as would induce everyone to wish an acquaintance with them. Besides this, the faculties of the mind, like the members of a body, are strengthened and improved by exercise. Mathematical reasoning and deductions are, therefore, a fine preparation for investigating the abstruse speculations of the law."
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> *Thomas Jefferson*[1]

I am not a lawyer; I have no formal training in the law, although I think I have a better-than-average amateur knowledge of the subject. I do not claim to be any kind of expert on any aspect of law. I am a mathematician. I have professional training and experience in the type of reasoning mathematicians and logicians (who I group with the mathematicians) use. My purpose in this essay is not to criticize or belittle anyone's mode of reasoning, but rather to explain how mathematicians reason in contrast to the predominant way of thinking in the law; I believe it is worthwhile for all of us to understand and appreciate different approaches to reasoning as a way of broadening our own thought processes.

I have always had a strong amateur interest in the law, particularly constitutional law. There are aspects of the law that have greatly perplexed me over the years, however. For a long time I thought that lawyers and judges were generally quite obtuse when it comes to logical reasoning. While I still think this about a few of them, I have rather recently come to realize that the "faults" in logic which I saw throughout the law and judicial decisions were not really faults at all, but instead simply a different way of thinking than I am used to. I also recognize that lawyers often try to spin the truth to the advantage of their clients, a separate matter which I will not discuss further, although I will not be as cynical as Benjamin Franklin[2]:

> I know you lawyers can, with ease,
> Twist words and meanings as you please,
> That language, by your skill made pliant,
> Will bend to favour every client;
> That 'tis the fee directs the sense
> To make out either side's pretence;
> When you peruse the clearest case,
> You see it with a double face,
> For skepticism's your profession;
> You hold there's doubt in all expression.

There is, of course, already a large literature on this subject, most of which I, as a nonexpert, am not familiar with. One classic reference I found fascinating reading, although representing a way of thinking rather foreign to my own, is Cardozo's *The Nature of the Judicial Process.*

I have identified two overriding differences in thought processes between mathematics and the law. These two may seem superficially similar, but I will describe each and try to distinguish them.

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[1] Quoted in *Cajori's Teaching and History of Mathematics in the U.S.* (Washington, 1890), p. 35.
[2] *Poor Richard: The Almanacks.*

1. Definitions.

"When I use a word, it means just what I choose it to mean – neither more nor less."

Humpty-Dumpty[3]

One of the most characteristic features of mathematics is that it really has its own language. A mathematics paper may superficially appear to be written in English (or French, Russian, etc.), but actually many of the words used are mathematical terms which have a precise meaning in mathematics often quite different from their nontechnical meanings. In fact, besides the abstractness of mathematics, the language of mathematics is one of the biggest obstacles mathematicians face in attempting to describe almost any nontrivial mathematics to a nonmathematician. (I have lapsed into math-ese here: "nontrivial" is one of our favorite words, and if you ever hear anyone say it, there is a good chance the person is a mathematician.)

Mathematicians carefully define every term they ever use (well, almost every term; a few fundamental terms like "point" and "set" are not defined), and, for ever after, each term means *exactly* what it was defined to mean, no more, no less, no different. For example, the word "group" has a specific meaning in abstract algebra, known to every mathematician. When a particular object is considered, there can be no difference of opinion about whether it is a group or not[4]; either it satisfies all the properties in the definition of a group, in which case it is a group, or it does not, in which case it is not a group. Mathematics could not hold together logically if we did not make such precise definitions.

Even the logical relationship of terms in mathematics is not necessarily the same as in general English. For example, in nontechnical English, *inequality* is the opposite of *equality*. But not in mathematics: a mathematical equality is often an example or special case of a mathematical inequality!

The situation is very different in the law. While the laws are full of definitions of various legal terms, the fact is that the vast majority of terms used in the law are not defined; they are taken to have their usual English meanings. (I should say that I am restricting attention to United States law in this essay, since I have little knowledge about the legal systems in other countries.) The "problem" is that terms used in the law may well have slightly different meanings to different people, or even to the same person at different times or in different contexts, and the same term can even be interpreted to have different meanings in different sections of law. Lawyers will argue that this "problem" is not a problem at all, but is in fact a strength of the law, leading to adaptability to changing social conditions. One could also argue, with considerable merit, that precise definitions in the mathematical sense of most terms in the law are not even possible; it is certainly much more problematic to try to give such definitions in the broad and all-encompassing realm of law and society in general than it is in the very restricted and well-delineated world of abstract mathematics.

Sometimes terms in the law are deliberately left undefined, especially at the level of the Constitution. A good example is the Fourth Amendment prohibition of "unreasonable searches and seizures." As I understand it, the authors of the Bill of Rights consciously declined to try to define what an "unreasonable" search is. Maybe they felt that they could not give an adequate definition covering all situations, or that it would be better to let the meaning evolve as the country matured and conditions changed.

Some legal terms have a general historical meaning, and no more specific definition in the law. An example is "bill of attainder." Courts have varied widely over the decades as to the exact meaning or applicability of the constitutional prohibition of bills of attainder, and many questions remain unresolved.

[3]Lewis Carroll (Charles Dodgson), *Through the Looking Glass*. It is well known that Dodgson was a mathematician.

[4]See the Appendix for a qualification of this statement.

The Thirteenth Amendment: Even terms which would seem to have widely accepted general meaning can lead to disagreements in specific situations. For example, consider the Thirteenth Amendment:

> "Neither slavery nor involuntary servitude, except as a punishment for crime whereof the party shall have been duly convicted, shall exist within the United States, or any place subject to their jurisdiction."

Most people would probably say that they have a pretty clear idea what this Amendment means. But do they? Specifically, what about military conscription (or, more accurately, compulsory registration for a military draft)? Does conscription constitute "involuntary servitude"? Different people will certainly disagree (I personally would have a hard time giving a definition of "involuntary servitude" which did not include military conscription, especially in light of the context, where the exception would seem to sanction confinement or forced labor for convicted persons). Discussion of this example will be continued later.

African-Americans: For another example, what is the definition of "African-American"? Everyone knows the history and rationale behind this term, and has a pretty good idea of who it is supposed to refer to, but as a mathematician I want a precise definition (by this I mean a definition which would be universally accepted and which will unambiguously specify whether or not any given person is "African-American"). I question whether such a definition is really even possible; for example, could you give a definition of the word *table* which is precise enough to unambiguously specify whether every object in the world is or is not a table?

This question came to mind in connection with a purported case some years ago (I never heard the outcome, or even whether the story was true). An applicant to Harvard Medical School put down that he was African-American. He was accepted, but when he came for orientation Harvard discovered he was white, and rescinded his acceptance on the grounds of fraud. He sued. Turned out that while he was born in the U.S., his parents had immigrated from someplace in Africa, presumably either South Africa or (then) Rhodesia, where his ancestors had lived for several generations, and he therefore asserted that he could legitimately claim to be "African-American."

While this claim may be considered flippant (indeed, what about the person who once claimed to be "Native American" since "I've lived here all my life!"), it does call into question whether, and how, the precise definition excludes such a person. And what about the two outstanding students we have had in my department, brother and sister, who are native-born Americans whose parents are Egyptian (their father is a professor at the University)? To the best of my knowledge they have never claimed to be African-American, but could they legitimately make such a claim?

It has sometimes been specified that "African-American" refers to ancestry from sub-Saharan Africa. But if this is the definition, how long must the ancestors have lived there, and how long ago could they have left? In fact, according to modern theories of human evolution, we are all "African-American" by this definition; some of our ancestors just left Africa longer ago than others. And many (perhaps most) Americans who would be universally recognized as African-American cannot document specific African ancestors.

If the definition really refers to a person's racial background, how is this to be determined? Appearance alone is obviously a totally inadequate criterion. We won't revisit the "one drop of blood" question, but how would even this criterion be unambiguously established? Would a person claiming to be African-American have to be certified by a DNA test, and exactly what test outcome would be acceptable?

Maybe the criterion is instead "slave ancestry." But then what about Barack Obama? While practically everyone would agree that he is "African-American" (except for a few people who apparently still think he is just African!), so far as I know he had no ancestors or relatives who were slaves (actually, it now appears that he may have some slave ancestry after all – on his *mother's* side!).

More specific terms can also defy a legal definition. For example, I am convinced there is no legally acceptable definition of "loitering" for the purpose of making it illegal (i.e. which is not just a license for harassing "undesirables"). The dictionary definition is something like "the act of remaining in a particular public place without any apparent purpose" which is clearly not legally acceptable. Apparent to whom? A person's purpose is almost always apparent to himself. And I do things all the time whose purpose is almost certainly not "apparent" to anyone else. Anyway, if this is the definition, how could a suspicious person be charged with loitering? If he is suspicious, then his purpose, even if illegal, is apparent. (The British apparently have a law prohibiting "loitering for the purpose of engaging in prostitution" which would be a complete logical *non sequitur*.) Along somewhat the same lines is police busting teenagers or young adults for making out in a car in a secluded area where the only ones who would see them and perhaps be offended are people, mainly police officers, who look for them. It would arguably be more appropriate to charge people who look for such couples with voyeurism. The same goes for police who try to entrap people by soliciting prostitution or gay sex. I'm not advocating that people should be able to have sex in public, but places with a reasonable expectation of privacy are another matter.

As a related matter, in Nevada, and probably other states, it is illegal to drive through private property "for the purpose of avoiding a traffic control device." How can a prosecutor possibly prove a violation of this law (i.e. prove the driver's purpose) without a confession? Note, of course, that *consequence* is not the same as *purpose*.

Ambiguity of Application: Ambiguity of application can even arise with some of the most widely-known phrases in the law. For example, what does "freedom of speech" or "freedom of the press" really mean? Everyone knows these freedoms are not absolute and were never intended to be, and the well-known examples such as crying "fire" need not be discussed here. But how about this: does the First Amendment give me the right to speak Spanish, or Urdu, or Navajo if I so choose? Does freedom of the press give me the right to publish a newspaper in any language I want? I believe it is generally accepted today that I do have these rights, except possibly when conducting official business with the government, although of course in the past some people, notably Native Americans, have been severely punished for speaking their ancestral language (and people are also still sometimes harassed by police for speaking Spanish). But Canadians, who also have a guarantee of freedom of speech, may have a somewhat different attitude; to some people in Quebec, for example, freedom of speech seems to mean that you can say almost anything you want, as long as you say it in French (to be sure, French-speaking Canadians have had legitimate grievances).

Taking this question a step farther, do I have the right to speak or write in code, perhaps (but not necessarily) to limit my message to its intended audience? This may seem like a different issue, but I think it is really not so different at all: speaking or writing in a language unintelligible to most listeners or readers is often criticized as using a code. Mathematicians could also be accused of talking to each other largely in code! The government does frequently accuse gang or terrorist suspects of communicating in code; of course, planning or organizing criminal activity is not protected speech, code or no code, but the fact that a code is allegedly used is sometimes itself taken as evidence of a crime. The government also attempts to prohibit internet communications using codes that the spooks at the NSA cannot decipher; and the FBI director recently complained about new smartphones using encryption methods which even the service providers cannot decode, making it impossible for anyone including the government to eavesdrop. A logical absolutist reading of the First Amendment might guarantee me the right to utter any stream of sounds I choose, or to print or send over the internet any string of 0's and 1's I choose, and if the string of sounds or characters is intelligible to someone, so be it; the restrictions on content should be exactly the same as for communications in plain English. Indeed, communication is a very subtle matter: intonation and timing, as well as gestures, facial expressions, and body language can often be as important a part of communication as the actual

words used. Shouldn't all of these fall under the umbrella of "speech" protected by the First Amendment? Surely communication in sign language is protected. After all, it is now universally recognized that the First Amendment applies to symbolic speech of very varied nature.

On the subject of symbolic speech, I recently read about a midwestern town which regularly arrests people, mostly minorities, for violating the town's ordinance prohibiting wearing baggy pants in public. Los Angeles also prohibits wearing "Zoot Suits" (apparently no longer enforced). And there was New York's notorious, and recently repealed, "walking while trans" law; similar laws are still on the books in several states, including California, and several states are now trying to ban drag completely. (I love Stephen Colbert's comment: "Our Founding Fathers did not create this country so men could wear frilly shirts, silk stockings, and powdered wigs.") Where does a state or locality get the right to tell people what style of clothes (or makeup, etc.) they may or may not wear, or how they may wear their hair? Isn't a person's appearance a personal, cultural, or fashion statement which is symbolic speech protected by the First Amendment? What about prohibiting head scarves or yarmulkes? And why are baggy pants or cross-dressing really any different? An argument *might* be made that localities can ban clothing leaving certain body parts visible, under the same authority as banning "pornography" (we will refrain from getting into this issue further), and they *might* also have the authority to ban such things as certain types of offensive printed messages on T-shirts, but how can they go beyond that?

Another example has recently arisen: the attempts in Congress to make it illegal to advocate or participate in a boycott of Israel over the issue of Palestinean rights. Such a proposal seems to me such a blatant violation of the First Amendment that no one would seriously consider it (although there has recently been a bad 8th Circuit Court decision on a related but slightly distinct issue). What right does the government have to tell me what I have to buy and who I have to buy it from? And if it is legal for individuals to boycott, how can it be illegal to advocate that people do it? After all, the right to peaceably assemble pretty clearly gives the right to work together to achieve political goals. (I suspect politicians advocating this law do indeed know it would be unconstitutional but still support it for political reasons.)

As a related issue, there are laws prohibiting advocating or organizing a general strike. How can this not be a violation of the First Amendment? How can the government tell privately employed people they cannot choose to stop working, and if not how can it be illegal to advocate that they stop? I have always liked the concept of a general strike as an ultimate response to extreme circumstances, when people get so fed up with the social machine that they refuse to continue to participate. But it is obvious why the power elite would want to prohibit them, since their position of power depends on ordinary people continuing to work maintaining the status quo.

Substantial Application: There are even instances where some legal term is fairly precisely defined, but a particular situation is held to "substantially" satisfy the definition even though it does not technically satisfy all parts of it. Such a conclusion would be unthinkable in mathematics. As the definition is commonly written, a mathematical object must satisfy nine axioms to be called a vector space (for this discussion, it's not important what they are; if you have taken a course on Linear Algebra you have probably seen them). Suppose we have an object that satisfies eight of the axioms but not the ninth – shouldn't we say the object "substantially" satisfies the definition and thus should be called a vector space? Any mathematician would find this suggestion ludicrous, and with good reason: such an object would fail to have many of the basic properties of vector spaces which make them so useful. More importantly, mathematicians could never prove anything about vector spaces if they didn't know which of the axioms they could assume are satisfied in all vector spaces and therefore can be used in proofs.

Interlude

Here's a logical conundrum: if a disbarred lawyer is arrested, can he represent himself in court? A specific provision of disbarment is a prohibition against representing anyone legally; on the other hand, it is a recognized constitutional right that anyone can represent himself in court. This is a legal version of the Barber Paradox: a barber in a small town shaves everyone who does not shave himself, and only those people. Then who shaves the barber? (A solution proposed by someone: the barber is a woman.)

I once heard a person described on TV news as a "former ex-felon." I considered this just another example of the degeneration of the language. But I once knew someone who could be accurately described as a former ex-felon: he was convicted of a felony and went to prison, and some years after he was released he got a pardon, so he was no longer an ex-felon. A more interesting question is: what if instead of a pardon it was ruled that he had been wrongfully convicted and was exonerated. Would he be a former ex-felon? Would he have actually been a felon, or would he have only been *thought* to be a felon because of the wrongful conviction?

If a person testifies under oath that he will never commit a certain action, and subsequently does it, is he guilty of perjury? Can't a person change his mind? At the very least, wouldn't the prosecution in a perjury case have to prove not only that he committed the action, but that he intended to commit it at the time of the testimony? This is not just a hypothetical question: historically, many public employees including University professors have been required to sign loyalty oaths as a condition of employment, and some were threatened with dismissal or even perjury charges years later if they committed acts which were perfectly legal but considered to be violations of the oath.

For the last interlude, we must put aside our passions about a monstrous crime, the Oklahoma City bombing, to consider a legal issue that arose from it. Timothy McVeigh, by general agreement, was a key defense witness for another person charged in the case. But he was executed before the other case came to trial. So the question is: how can a defendant possibly have a fair trial if the government can execute his defense witnesses before the trial? (My personal resolution is simple: the government should not execute anyone.) And what about deporting witnesses?

2. Meaning of Statements.

> "It is generally understood that mathematicians concern themselves with objects or concepts about which they establish theorems by applying logical, preferably irrebuttable, arguments. This last detail ... has always scared the majority of people, accustomed as they are to produce and hear daily, even in exalted intellectual activities, tens of arguments each more contestable than the other: life in society would be impossible if everyone had to provide incontrovertible proofs of his assertions and to express himself clearly and without ambiguity."
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> *Roger Godement*[5]

[5] *Analysis I*, Springer-Verlag, 2004, p. 1.

The other major difference I see between mathematics and the law concerns how literally statements are interpreted. Some statements in law are just flat-out commonly interpreted to mean something quite different from what they actually say. A notable example is the Second Amendment. I believe practically every logician, including myself, would say that the meaning commonly attributed to it today is a grotesque misreading of the language or intent of the statement. (I hesitate to bring this issue up at all and will say no more about it.)

In mathematics, every statement means exactly what it says, no more, no less, no different. If I read a mathematical statement, I usually know exactly what it means; the only obstacle is that I may not know the definitions of some terms used in the statement. But once I look up the definitions of the terms, I know exactly what the statement means. Furthermore, any other mathematician reading the statement and knowing the definitions of the terms would also know exactly what the statement means, and would have exactly the same understanding of its meaning that I have. [NOTE: this does not mean that either of us knows whether the statement is *true*; we may or may not (and there are even mathematical statements which can be neither proved nor disproved, and for which it does not even make sense to ask whether they are "true" or "false").] Mathematics would descend into chaos if we were not able to attach a precise meaning or interpretation to every statement. For example, if I prove a statement about (all) groups, and a particular group comes under consideration, no one can argue that the statement does not apply to *that* group.

The situation is, again, very different in the law. Even if two people agree on the meaning of every term in a section of the law, they can very easily disagree on the meaning or applicability of the whole section. For example, let us return to the Thirteenth Amendment question we discussed earlier. Suppose for the sake of argument that we agree that military conscription is "involuntary servitude". A logician would then say it is a necessary conclusion that the Thirteenth Amendment prohibits military conscription, end of story. But such a conclusion would be far from obvious to a judge. Judges do not always interpret laws literally. A judge in a case like this might also consider such questions as the intent of the authors of the Amendment or the legislators who passed it, what the common understanding of the question is among the public, relevant legal precedents or previous opinions of judges or scholars, and perhaps even whether prohibiting conscription is wise public policy.

I am aware that the Supreme Court ruled in 1918 that military conscription is legal, based on the constitutional authority of Congress to declare war and raise an army. But the government, including the Supreme Court, had an almost nonexistent regard for civil liberties in general in 1918, and I regard it as a grotesque extrapolation to use the war and army powers as justification for a military draft. The Constitution also gives Congress the power to establish post offices; does this mean the government has the power to draft people to work in the post office? In fact, the war and army powers, as well as the powers to run post offices, issue money, collect taxes, etc., were merely centralizations of matters which had been left to the states under the Articles of Confederation.

An interesting related logical question concerns the relation between the thirteenth and fifteenth amendments. The "except as punishment ..." clause of the thirteenth amendment was clearly intended to allow the government to continue to sentence convicted persons to imprisonment or forced labor (in fact I cannot think of any other possible rationale for the clause, which has of course led to considerable abuse, a different issue); thus imprisonment or forced labor fall under the umbrella of "involuntary servitude." Then the fifteenth amendment prohibits denial of the right to vote on the grounds of "previous condition of servitude" (of course clearly intended to mean former slaves). Combining the two statements would seem to prohibit denial of the right to vote on the grounds of previous imprisonment, although many states do in fact deny ex-felons the right to vote. (In fact, I cannot even see constitutional grounds to prohibit currently incarcerated people from voting!)

Incidentally, when are the "right-to-life" people going to argue that the Thirteenth Amendment prohibits

imprisoning a pregnant woman, since this would imprison a person (the fetus) not convicted of anything? (I'm not holding my breath.)

The Fourteenth Amendment: Perhaps a clearer example comes from the Fourteenth Amendment. This is a long Amendment; the relevant statement is

> "All persons born or naturalized in the United States, and subject to the laws thereof, *are citizens* of the United States and the state in which they reside." (emphasis added)

This statement always seemed to me so straightforward and unequivocal that even a lawyer should be able to understand it. *Even a Supreme Court justice* should be able to understand it. So I have long been mystified by the decision in a citizenship case (*Perez vs. Brownell*, 1958). Perez was born in Texas, but lived most of his life in Mexico and represented himself as a Mexican national while working in the U.S. on several occasions. He did not register for the U.S. draft during World War II, and remained in Mexico during the latter part of the war, allegedly to avoid the U.S. draft. He voted in the Mexican election in 1946. In 1952 he returned to the U.S. and claimed citizenship by birth, but the government argued that, under the Nationality Act of 1940, he had forfeited his citizenship. This act provided that the citizenship of a native-born person would be revoked for, among other things, either voting in a foreign election or remaining outside the U.S. during a declared war or national emergency to avoid military service. Perez argued that Congress had no power to enact such a law, but the Supreme Court disagreed and upheld the constitutionality of the law (under the theory that Congress has the inherent authority to regulate foreign affairs) and the revocation of citizenship. There was no evidence that Perez had ever renounced his U.S. citizenship; at most, he was asserting dual citizenship (he presumably qualified as a Mexican citizen under Mexican law). Earl Warren wrote a forceful and eloquent dissent; amazingly, William Brennan concurred with the majority.

I never understood until recently how a person presumably intelligent enough to be appointed to the Supreme Court could read the statement in the Fourteenth Amendment and not conclude that Perez was a U.S. citizen by birth, period. As a logician would read this statement, there is not even a provision for a person to voluntarily renounce citizenship by birth, let alone have it involuntarily revoked. (This interpretation is not as zany as it sounds. Many countries do not allow voluntary renunciation of citizenship, and regard such persons as still being citizens for tax purposes or compulsory military service, for example.) The only logical qualification would be if there was a contradictory phrase elsewhere in the Constitution; but a provision in an amendment must take precedence over anything to the contrary in the body of the document (or earlier amendments), especially something held to be inherent but not explicitly stated. I now understand, however, that a statement such as the one in the Amendment is not just to be taken literally, but interpreted in the context of a broad range of factors, although I still regard the decision as a bad one; it should be noted that the Supreme Court eventually agreed and overruled *Perez vs. Brownell* less than ten years later, in 1967 (*Afroyim vs. Rusk*).

This issue has again arisen through the claim of the legal scholar who again occupies the White House that he can eliminate "birthright citizenship" by executive order. Any lawyer who told him this was possible (I doubt there were any, probably not even Rudy or Sidney Powell) should be disbarred. But almost as baffling to me are the number of people, including some legal scholars, who say that Congress could limit it by legislation. My firm opinion is that no reasonable person could think it could be done by anything short of a constitutional amendment; any other interpretation of the Fourteenth Amendment language is patently illogical (of course, this didn't stop Congress from ignoring the language in 1940 with the Nationality Act, or even the Supreme Court in 1958; and despite the clear wording, Native Americans were not generally recognized as citizens until the Indian Citizenship Act of 1924, and did not universally have the right to vote

until 1930 despite the 15th Amendment). Note that this is not the first time a country has tried to eliminate birthright citizenship: South Africa, during the latter years of apartheid, did something similar, trying to strip citizenship from its Black population by creating fictitious "homelands" to which troublemakers could be "deported" (anyone remember the country of Bophuthatswana?)

The Reserve Clause: Another example, in which the reasoning is more complex and less clear-cut, is the former baseball "Reserve Clause." For decades, the standard professional baseball player's contract was a one-year deal, which contained the following provision, known as the Reserve Clause:

> "... the Club shall have the right by written notice to the Player at said address to renew this contract for one year on the same terms, ..."

(The omitted language concerned timing and salary; the clause could be invoked if the club offered and the player declined to sign a new contract by a certain date.)

The position of the owners was that if the contract was renewed for one additional year "on the same terms," then the Reserve Clause was also part of the renewed contract, and the Club therefore had the right to renew the contract again for a second year and, theoretically, in perpetuity, in effect binding the player to the Club for life, or at least as long as the Club wanted him. This interpretation was widely criticized, but never effectively challenged until 1975.

In 1975 Andy Messersmith declined to sign a new contract and played the season under his 1974 contract, renewed under the Reserve Clause. He again declined to sign for 1976, and the club asserted its right to again renew the 1974 contract. Messersmith, one of the leading pitchers of the time, claimed that the club had exhausted its one-year right of renewal, and that the contract had thus expired, making him a free agent. The matter was sent to binding arbitration, and the arbitrator ruled in favor of Messersmith, declaring him a free agent and effectively ending the Reserve Clause and ushering in the free agent era in baseball. (There were some other contemporaneous cases which were part of the story, notably Dave McNally and Curt Flood, but the Messersmith case was the cleanest and most influential from a logical standpoint.)

I believe it is a general consensus of legal experts that the arbitrator's decision was correct from a legal standpoint. If so, I would certainly not take issue; I was personally pleased by the ruling, and I regarded the Reserve Clause as an odious example of restraint of trade. However, I must acknowledge that from a purely logical standpoint, the owners' interpretation of the meaning of the Reserve Clause language was probably correct.

Rounding Numbers: While on the subject of baseball, let me digress briefly to another situation having nothing to do with law, but which illustrates a contrast in thinking between mathematicians and the public. Going into the final day of the 1941 season, Ted Williams had gotten 179 hits in 448 at bats, a batting average of $.39955\cdots$, which rounds off to .400 to three decimal places. The Red Sox were scheduled to finish the season with a doubleheader, and with nothing in the standings at stake the Red Sox manager suggested that Williams sit out to "preserve" his .400 average. If this had happened, it appears that the public would have accepted him as a .400 hitter. But few mathematicians would have agreed: whatever it rounds off to, $\frac{179}{448} = .39955\cdots$ is a number *less* than $.400 = \frac{2}{5}$ (funny how people can think gas at $\$3.99\frac{9}{10}$/gal costs less than \$4/gal, but a man who bats $.39955\cdots$ has hit .400). Fortunately, Williams refused to sit out, and removed all doubt by going 6 for 8 in the doubleheader to raise his final season average to .406 .

Note that while it is customary to round baseball batting averages to three decimal places, there is nothing requiring this in the baseball rules. There have been batting titles decided in the fourth decimal place, notably in 1949, ironically involving Williams, who hit $.342756\cdots$ that year. George Kell of Detroit

batted .342911 $\cdots$. Although both averages round off to .343 to three places, Kell was recognized as the American League batting champion and it was not regarded as a tie. (There was an even closer batting title in 1945, but those averages rounded off differently to three places.) It is even theoretically possible for a batting title to be decided in the *sixth* decimal place, for example if one batter goes 199 for 598 and another 200 for 601 (these numbers are within established performance levels by everyday players).

Rounding off numbers leads to interesting and sometimes complex mathematical problems. For example, allocating seats or delegates in a political body is really a number roundoff problem; for an interesting account of the problems and the history of apportioning seats in the U.S. House of Representatives, see the book *Fair Representation: Meeting the Ideal of One Man, One Vote*, by M. Balinski and H. P. Young. For another simple example of a number roundoff problem leading to a potential disagreement between mathematicians and the public, back during the national 55 MPH speed limit there was a question on the *Jeopardy* TV show: "If you drove 90 kilometers per hour in a 55 MPH zone, would you be speeding?" The "correct" answer given on the show was: No, since 90 kilometers per hour is "almost exactly" 55 miles per hour. I took strong exception to this answer, since in fact 90 kilometers is *more* than 55 miles (it is about 55.4), although I acknowledge that the possibility is remote that someone would be cited for exceeding the speed limit by such a slight amount, or that a prosecutor could prove beyond a reasonable doubt that such a person was actually speeding since .4 MPH is well within the accepted margin of error of police radar systems.

Literal Interpretation: Getting back to our central subject, the law, sometimes a literal interpretation of a legal document leads to a situation unforeseen by the creators of the document, and in a situation like this it is arguably more fair to interpret the language less than literally and more in line with the intentions of the writers. An example was a contract which led to a lawsuit in which one of my colleagues in the mathematics department was called as an expert witness (to in effect testify about negative numbers), since the clause in question actually involved a mathematical formula. The Wells Rural Electric Company contracted with an Idaho power company to build a transmission line, over which Wells Rural Electric would buy power for a specified period of time. The contract contained an escape clause: Wells could cancel the contract by paying a penalty of a fixed dollar amount "less the salvage value of the line." Wells did cancel the contract several years later, but in the meantime the scrap price of copper had increased so much that the salvage value of the line was more than the fixed penalty figure, so the formula yielded a negative penalty; Wells then went to court to try to recover the difference from the Idaho company. I don't recall that I ever heard the outcome of the case, and I have no idea how the law is normally interpreted in such a case. I imagine that there is a legal convention in situations like this, but I also know that in many instances, such as in tax law, the problem of potentially negative numbers is anticipated and explicitly provided for in the language of the document.

During the testimony of one of Nixon's Watergate henchmen, a poorly phrased question led to a logical quagmire. The witness was asked: "Is it your testimony that ...?" The part after the ... was a statement which was false and known to the witness to be false. He answered "Yes." When he was charged with perjury, he claimed he had in fact answered the question correctly and truthfully, since the question asked not whether the statement was true but merely whether it was his testimony! I don't think there is a clear logical answer here; reasonable logical arguments can be made either way. But on balance, as a logician I would have to hold my nose and admit that the witness was technically correct. (I doubt this is correct from a legal standpoint.) A crucial distinction, of course, is whether this question was the entire exchange on the subject or whether it was a follow-up to a previous statement by the witness. If it was a stand-alone question, is it really essentially different from asking the witness under oath, "Have you claimed that ...?" If the witness had in fact previously made that claim, and the claim was false, would it be perjury for him to answer "Yes"? (a "No" answer would seem to be perjury in this case.) What if the previous false claim had

not been made under oath? (Another of the henchmen, when confronted with contradictions in his story at the Watergate hearings, uttered the classic line: "I stand by my previous testimony, whatever that was.")

Even seemingly ironclad principles in law are not so ironclad when examined carefully. Consider, for example, the well-known principle "ignorance of the law is no excuse." This principle is not nearly as universal as one might at first think. Suppose, for example, a state decided to do away with stop signs, speed limit signs, and no parking signs, and just passed an enormous law specifying which intersections in the state a driver must stop at, what the speed limit was on every street and road in the state, and precisely where in the state a driver may or may not park. Such a law would create an impossible mass of information for even a local driver, much less a visitor, to digest and memorize, would create chaos on the roads, and would be unenforceable since no judge could reasonably expect a driver to know everything in the law; "ignorance of the law" might in fact be a reasonable defense, especially if the driver is in a location unfamiliar to him. Under our traffic law system, with good reason, a driver cannot be cited for failing to stop at an intersection or exceeding a low speed limit unless there is a sign present to that effect. (The law can, and does, list a manageably small number of specific situations where a driver must stop, such as for a school bus, and sets a default speed limit in effect when there are no signs to the contrary posted.)

I'm sure that as a practical matter if nothing else, no state would consider doing this, since the law would have to be amended every time road conditions changed; but it is an interesting theoretical exercise. A possible variation poses some interesting questions: instead of specifying all the details in the law, suppose the law just required drivers to conform to requirements listed in a DMV database which could be regularly updated. Would unfamiliarity with the current details of such a database be reasonably considered "ignorance of the law"? (One could imagine that at some time in the future it might be expected that every vehicle have an electronic device which would access and keep track of such details.)

On the subject of road signs, there are a great number of poorly written, even incomprehensible, road signs around the country (I have posted some of the most outrageous ones I have seen in the Photo Gallery). One common example I take issue with is that a school speed limit is posted and in effect "when children are present." I never know what such a sign means: specifically, what does "present" mean? Present on the street? (Does this mean "visible on the street"? How close to the street?) Present in a car (yours or someone else's)? Present in the school? Present in the community? How could a driver ever know for sure at any hour of the day or night, even at 2 AM Sunday, that children were not "present"? I'm not opposed to school speed limits, but they need to be comprehensible.

Flag-Burning: A final (hopefully hypothetical) illustration of a range of issues involving both differences I have described between mathematics and the law is the so-called "flag-burning" amendment which was mercifully (but uncomfortably closely) rejected in Congress several years ago. The entire text of this proposed amendment reads:

> "The Congress shall have power to prohibit the physical desecration of the flag of the United States."

Suppose, God forbid, this amendment ever passes and goes into effect, which would be a sad day for this country, representing the first official weakening of the Bill of Rights in our history. To a logician, and maybe to a lawyer, though, this situation would spawn a whole range of interesting questions. To begin with, what does "physical desecration" mean? Would it include, for example, sewing a flag on the seat of one's pants? What about posting a film clip or cartoon on the internet depicting the burning of a flag?

Here is another interesting logical conundrum. Suppose the proposed amendment passes, and Congress considers legislation prohibiting flag-burning. In an attempt to defeat the legislation, opponents organize a

pledge campaign of mass civil disobedience to take place if the law goes into effect, threatening to clog the legal and prison systems with flag-burners protesting the law. Could the organizers of such a campaign be charged with conspiracy? Can one conspire to violate a law which has not yet been enacted? Isn't that an *ex post facto* situation? (Note that the amendment itself does not prohibit flag-burning; it only authorizes Congress to pass legislation prohibiting flag-burning.)

My main point relative to this situation, though, concerns the question of what constitutes an American flag. We all know that our society is full of objects which resemble American flags to a greater or lesser degree, from objects which are just red, white, and blue, adorned with stars and/or stripes, only vaguely resembling an actual flag, to objects closely and intentionally resembling an actual flag but with, say, the stars replaced by a peace symbol. Do any or all of these objects themselves represent a "physical desecration" of the American flag? More to the point, what if such an object is burned in protest?

The flag of the United States is quite precisely defined or described in the law: its relative dimensions, its colors, and the number and arrangement of stars and stripes. In passing legislation pursuant to the amendment, would Congress be limited to prohibiting "desecration" of only objects exactly meeting the legal definition of the American flag, or could legislation broadly prohibit "desecration" of objects resembling the American flag? Could Congress adopt a broader definition of the "flag of the United States" in this legislation than in the law specifying the design of the flag? For that matter, could Congress significantly broaden the general definition of the American flag to include just about anything that is red, white, and blue, and have the amendment apply to such objects?

Suppose the definition of the American flag in the law is not changed from the current one. Then, in any prosecution of a person for allegedly burning a flag, wouldn't the prosecution have to prove that the object burned exactly met the legal definition of an American flag? Wouldn't burning an object resembling a flag but with, say, 51 stars, still be protected by the First Amendment?

I know how a mathematician would answer these questions. I would be very interested to find out how lawyers and judges would answer them (although I hope never to see the day when these questions need to be addressed in court decisions!)

> "The pursuit of mathematical science makes its votary appear singularly indifferent to the ordinary interests and cares of men. Seeking eternal truths, and finding his pleasures in the realities of form and number, he has little interest in the disputes and contentions of the passing hour. His views on social and political questions partake of the grandeur of his favorite contemplations, and, while careful to throw his mite of influence on the side of right and truth, he is content to abide the workings of those general laws by which he doubts not that the fluctuations of human history are as unerringly guided as are the perturbations of the planetary hosts."
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> *Thomas Hill*[6]

Appendix

It is actually not true that there can never be a difference of opinion among mathematicians over whether a particular mathematical object is, say, a group. There are several properties which a group must have, and in some cases it is not obvious whether the properties are satisfied in a particular example; mathematicians

[6] "Imagination in Mathematics," *North American Review*, Vol. 85, p. 227.

may very well have different opinions about whether the properties hold in the example. Perhaps a better example is that mathematicians frequently disagree whether some unproved mathematical statement is true.

However, such a disagreement is very different in nature from a disagreement about whether military conscription constitutes "involuntary servitude", for example. The mathematical disagreement is not really a difference of *opinion*, but rather a difference of *prediction*. In its nature, if not its significance, it is more like an argument between Terry Bradshaw and Howie Long on the Pregame Show over which team will win the game. Everybody knows that in a few hours everyone, including the disputants, will know which one was right and which was wrong, and the argument will be definitively settled to the agreement of all parties. In the mathematical case, there may not be such a definite time frame, but the disputants know it is likely that eventually someone will prove one of them right and one wrong, and both will accept the conclusion.

Such a resolution is not possible in a true difference of opinion such as the military conscription example. The *legal* question may (or may not!) be definitively settled by a court decision, but the disagreement will not be settled; one of the disputants will very likely regard the court decision as wrong, and the disagreement will continue.

The concept of "wrongly decided," common in legal discussions, is totally foreign to mathematics. The only mathematical situation even vaguely similar is that occasionally someone claims to have proved a statement, and a flaw (error or gap) is found in the proof. In this case, mathematicians rarely use the phrase "wrongly proved" which is actually a logical *non sequitur* (either something is proved or it is not); the common description is simply "not proved." (The phrase "wrongly analyzed" can occasionally be used, especially in applied mathematics, but it does not necessarily imply a mathematical mistake; it might just mean that the mathematical results obtained do not accurately reflect the problem they were meant to resolve.)